Transient Solutions to a Two-Country Dynamic Model Considering Work Skills and Migration

WRSA 2019
Asao Ando, Tohoku University
Tomohiro Yokoyama, Tohoku Electric Power Co.

http://www.geocities.jp/ando_sdj/

Coexistence of economies with growing and stagnating populations

Per capita GDP (left in US$) and Population (right in millions) for selected countries (Source: WDI)

		2000		2015		Growth rate (% p.a.)	
Meddle income countries	China	954.6	1260	7924.7	1370	15.2	0.56
	Vietnam	433.3	77.6	2111.1	91.7	11.1	1.12
	Philippines	1039.7	77.9	2899.4	101	7.08	1.75
OECD countries	Spain	14787.8	40.3	25831.6	46.4	3.79	0.94
	Italy	20051.2	56.9	29847.0	60.8	2.69	0.44
	Japan	37299.6	127	32477.2	127	-0.92	0.00

Developing economies are growing with growing population.

Drastic economic growth cannot be expected for developed economies with stagnating population. → For a developed economy to make a soft landing to a new equilibrium state, relationship with developing economies is important.

http://www.geocities.jp/ando_sdj/

Existing dynamic models of economic growth

Solow model: Balanced growth path with fixed per capita physical capital *k*.

Control the savings rate *s* to achieve maximum utility *U*(*c*).

- Point economy with uniform workers.
- Dynamics is concerned with the stability of the converged equilibrium against perturbations

Ramsey-Cass-Koopmans model:

Optimal control to maximize the present value of future utilities

$$\max_{C(t)} U = \int_{t=0}^{\infty} e^{-\delta t} \frac{C(t)^{1-\theta}}{1-\theta} \left(\frac{L(t)}{H} \right) dt$$
$$s.t. \int_{t=0}^{\infty} e^{-R(t)} C(t) \frac{L(t)}{H} dt \leq \frac{K(0)}{H} + \int_{t=0}^{\infty} e^{-R(t)} W(t) \frac{L(t)}{H} dt$$

- Point economy, but distinct households and workers.
- Dynasty model of Infinite time horizon, but the major concern is convergence to the steady state.

➢ The dynamic path towards the optimum could include a series of *temporal disequilibria*.

➢ Existing dynamic models are mostly concerned about a closed economy, and cannot handle the stability with international trade. (Richard et all. 2005)

Motives for the model

Economies are not always steady.

- Even if the steady state exists, the path to reach it does *not always exist*.
- To study the transient path to achieve the long-run social optimal.

Co-existence of population growing *South* and population declining *North*.

- The world include economies on the different stages of development.
- They interact through trade, financial flows, and migration.
- Significance of human capital formation through education.

➢ The new equilibrium with fewer population may be desirable; The goal of development is not GDP, but per capita income or individual utility.

➢ Search for the region of parameters which keep the economies sustainable throughout the planning horizon.

This study also reflects master theses by former students; Harigaya (2005) and Li (2007).

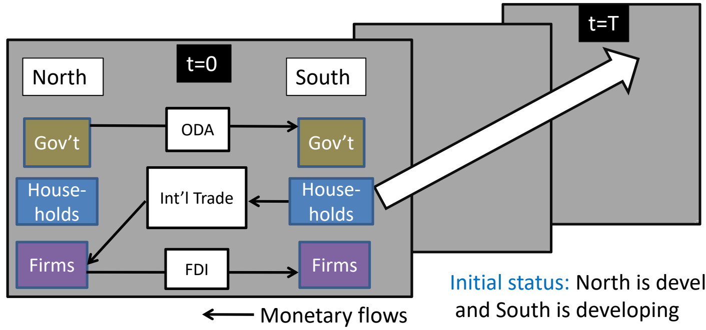
Outline of the model

Households	Firms	Government
Consumption, saving, and tax payment	Production and physical investment	Human capital investment (Education)

Production and human capital	
Production	Both countries produce the same numeraire good Y_j
Factors of production	Physical capital K_j, Skilled workers H_j, Unskilled workers L_j
Skill development of workers (Education)	Unskilled workers L_j can become skilled H_j through education at the rate of Ψ persons per annum.
Migration of workers	Skilled workers H_j can move to the country with higher per capita income at the rate of MH persons per annum.
Birth	Fertility rate g_j is fixed regardless of skills. Everybody is born unskilled.
Death	Common mortality rate δ_j applies to both L_j and H_j. (Markov)

Balance of payment	ODA	FDI
Trade surplus NX is fully distributed as ODA and FDI	Deficit country allocates ξNX to education	Deficit country allocates (1-ξ) NX to physical investment

Dynamics of transactions between two economies

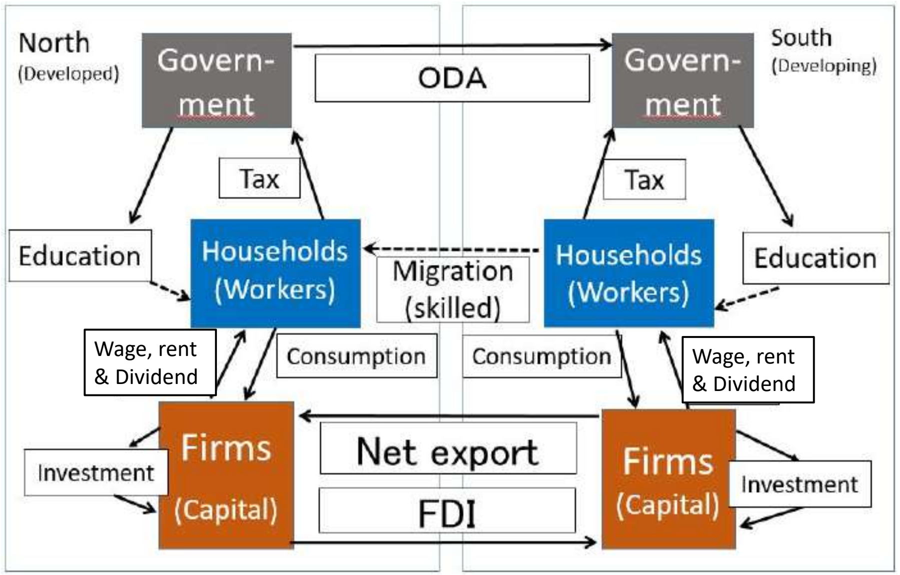


Initial status: North is developed and South is developing

Optimal control problem with finite horizon (T periods)

- The social welfare is the present value of Benthamite utilities.
- The world government problem to maximize the weighted sum of the social welfares in two countries (as the reference point).
- Search the set of parameters to reach the terminal status (existence).

Details of instantaneous transactions



The firms

Product	Factors of production
Numeraire (Y_j)	Physical capital (K_j), Skilled workers (H_j), Unskilled workers (L_j)

$$Y_j(t) = f\left(K_j(t), H_j(t), L_j(t)\right)$$

<u>Profit π</u>

$$\pi_j(t) = Y_j(t) - r_j(t)K_j(t) - w_{Hj}(t)H_j(t) - w_{Lj}(t)L_j(t)$$

where r : capital rent and w : wage. j indicates the country (N or S).

<u>First order condition for profit maximization</u>

$$\frac{\partial f_j}{\partial K_j} = r_j(t), \frac{\partial f_j}{\partial H_j} = w_{Hj}(t), \frac{\partial f_j}{\partial L_j} = w_{Lj}(t)$$

The workers

Birth	Fertility rate g_j is fixed in each country (regardless of skills).
Capacity building	Unskilled worker (L) can be educated to become skilled (H). The efficiency of education Ψ is a function of the government budget G.
Migration	Only skilled workers are allowed to migrate to the country with higher income. The speed of migration MH is a function of income difference.
Death	The mortality rate δ applies to workers (regardless of skills).

The government is specialized in education; its budget $G_j(t)$ is financed by tax revenue and ODA.

State equations for the workers:

$$\dot{H}_j = -\delta_j H_j(t) + \Psi\left(G_j(t)\right) + MH_j(t)$$

$$\dot{L}_j = g_j\left(H_j(t) + L_j(t)\right) - \delta_j L_j(t) - \Psi\left(G_j(t)\right)$$

Speed of migration for skilled workers:

$$MH_1(t) = -MH_2(t) = \eta\left(c_{H1}(t) - c_{H2}(t)\right)$$

Households (workers)

- Firms and physical capital are owned by the domestic workers:
- Profit π_j + capital rent $r_j K_j$ are distributed to the domestic workers
- Skilled workers own θ_j times as much shares than unskilled workers.
- They must give up present shares when migrating (to avoid hysteresis).

Control variables	
Propensity to save	s_j
Tax rate	T_j

Other endogenous variables	
Per capita consumptions	c_{Hj}, c_{Lj}
Wages	w_{Hj}, w_{Lj}

Per capita income of skilled workers

$$y_{Hj} = \left(\frac{\theta_j\left(\pi_j(t) + r_j(t)K_j(t)\right)}{\left(\theta_j H_j(t) + L_j(t)\right)} + w_{Hj}(t)\right)$$

Per capita income of unskilled workers

$$y_{Lj} = \left(\frac{\pi_j(t) + r_j(t)K_j(t)}{\left(\theta_j H_j(t) + L_j(t)\right)} + w_{Lj}(t)\right)$$

	$y_{Hj}*H_j$	$y_{Lj}*L_j$
Tax revenue T_j	τ_j	τ_j
Total saving S_j	$s_j(1-\tau_j)$	$s_j(1-\tau_j)$
Consumption C_j	$(1-s_j)(1-\tau_j)$	$(1-s_j)(1-\tau_j)$

Balance of payments

Balance of payments=Trade balance + Transfer balance + Financial balance

= NX - ODA - FDI

current account non-trade balance

	Behavior	Trade balance	Financial flows
Developing economies	Develop with foreign debt	−	+
Developed economies	Invest trade surplus to foreign countries	+	−

In the two-country world, one country has trade surplus, the other has deficit.

- Country with surplus→Divide the surplus to ODA and FDI to invest in the other country.
- Country with deficit→Use ODA to supplement government spending and FDI to enhance physical capital.

➢ The fund received by the deficit country is accumulated. (ODA includes loan and grant.)
➢ Expected to repay all the debts (principal and interest) to creditor by the terminal year.
➢ In the later years, the positions of surplus and deficit countries necessarily exchange.

ODA/FDI and human/physical capital formations

$$NX_j(t) = \xi(t)NX_j(t) + (1 - \xi(t))NX_j(t)$$

Trade surplus in N ODA to S FDI to S

$$0 \le \xi_j(t) \le 1$$

$$NX_N = -NX_S$$

Government	Specialized in human capital formation

$$G_j(t) = T_j(t) - \xi(t)NX_j(t)$$

Tax revenue ODA

$$\dot{H}_j = -\delta_j H_j(t) + \Psi\left(G_j(t)\right) + MH_j(t)$$

Firms	Physical capital formation in both countries

$$I_j(t) = S_j(t) - \left(1 - \xi(t)\right)NX_j(t) \longrightarrow \dot{K}_j = I_j(t) - \rho_j K_j(t)$$

Savings FDI

Cumulative debts

ODA and FDI are accumulated with the fixed interest rate r_p.

Cumulative debts as the state variables

$$A_O(t) = \int_0^t e^{-r_p t'} \xi(t') NX_j(t') dt'$$

$$A_F(t) = \int_0^t e^{-r_p t'} (1-\xi(t')) NX_j(t') dt'$$

State equations

$$\dot{A}_O = e^{-r_p t} \xi(t) NX_j(t)$$

$$\dot{A}_F = e^{-r_p t} (1-\xi(t)) NX_j(t)$$

Initial and terminal conditions

$$A_O(0) = 0,\ A_O(T) = 0$$

$$A_F(0) = 0,\ A_F(T) = 0$$

The world government problem

$$\max_{s_1, s_2, \tau_1, \tau_2, \xi, NX} W = \sum_{j=1}^{2} \omega_j \left\{ \int_0^T e^{-it} \{ u(c_{Hj}(t)) H_j(t) + u(c_{Lj}(t)) L_j(t) \} dt \right\}$$

ω: weight, i: subjective discount

State equations

$$\dot{H}_j = -\delta_j H_j(t) + \Psi(G_j(t)) + MH_j(t)$$

$$\dot{L}_j = g_j(H_j(t) + L_j(t)) - \delta_j L_j(t) - \Psi(G_j(t))$$

$$\dot{K}_j = I_j(t) - \rho_j K_j(t)$$

$$\dot{A}_O = e^{-r_p t} \xi(t) NX_j(t)$$

$$\dot{A}_F = e^{-r_p t} (1-\xi(t)) NX_j(t)$$

Control constraints

$$0 \le s_j \le 1, 0 \le \tau_j \le 1, 0 \le \xi_j \le 1$$

$$G_j(t) = T_j(t) - \xi(t) NX_j(t) \ge 0$$

$$I_j(t) = S_j(t) - (1-\xi(t)) NX_j(t) \ge 0$$

Initial and terminal conditions

$$A_O(0) = 0,\ A_O(T) = 0$$

$$A_F(0) = 0,\ A_F(T) = 0$$

The Hamiltonian function

$$\tilde{H} = e^{-it} \sum_{j=1}^{2} \omega_j \left(u(c_{Hj}(t) H_j(t)) + u(c_{Lj}(t) L_j(t)) \right)$$

$$+ \lambda_{11}(t) \dot{K}_1 + \lambda_{12}(t) \dot{K}_2$$

$$+ \lambda_{21}(t) \dot{H}_1 + \lambda_{22}(t) \dot{H}_2$$

$$+ \lambda_{31}(t) \dot{L}_1 + \lambda_{32}(t) \dot{L}_2$$

$$+ \lambda_4(t) \dot{A}_F + \lambda_5(t) \dot{A}_O$$

$\dot{K}$: Change in physical capital stock

$\dot{H}$: Change in skilled workers

$\dot{L}$: Change in unskilled workers

$\dot{A}_F$: Change in FDI related cumulative debts

$\dot{A}_O$: Change in ODA related cumulative debts

The Lagrangian function

$$\tilde{L} = \tilde{H}$$

$$+ \sum_{j=1}^{2} \begin{pmatrix} \mu_{1j}(1 - s_j(t)) + \mu_{2j}(1 - \tau_j(t)) \\ + \mu_{4j}\left(s_j(t)(1-\tau_j(t)) f(K_j(t), H_j(t), L_j(t),) - (1-\xi(t)) NX_1(t)\right) \\ + \mu_{5j}\left(\tau_j(t) f(K_j(t), H_j(t), L_j(t),) - (1-\xi(t)) NX_1(t)\right) \end{pmatrix}$$

$$+ \mu_3(1 - \xi(t))$$

With the following control constraints being considered.

$$0 \le s_j(t) \le 1, 0 \le \tau_j(t) \le 1 \rightarrow \mu_1, \mu_2$$

$$I_j(t) \ge 0, G_j(t) \ge 0 \rightarrow \mu_4, \mu_5$$

$$0 \le \xi_j(t) \le 1 \rightarrow \mu_3$$

Optimal conditions

λ: costate variables

→marginal values of corresponding state variables

$$-\dot{\lambda}_{11}=\frac{\partial\tilde{L}}{\partial K_1},-\dot{\lambda}_{12}=\frac{\partial\tilde{L}}{\partial K_2}$$

$$-\dot{\lambda}_{21}=\frac{\partial\tilde{L}}{\partial H_1},-\dot{\lambda}_{22}=\frac{\partial\tilde{L}}{\partial H_2}$$

$$-\dot{\lambda}_{31}=\frac{\partial\tilde{L}}{\partial L_1},-\dot{\lambda}_{32}=\frac{\partial\tilde{L}}{\partial L_2}$$

$$-\dot{\lambda}_{4}=\frac{\partial\tilde{L}}{\partial A_F}=0,-\dot{\lambda}_{5}=\frac{\partial\tilde{L}}{\partial A_O}=0$$

$$\lambda_{ij}(t+1)=\lambda_{ij}(t)+\dot{\lambda}_{ij}(t)$$

→Numerically calculate the state variables in the succeeding period.

Convergence is judged from the value of state or costate variables in the terminal time.

$$\lambda_1(T)=0,\lambda_2(T)=0,\lambda_3(T)=0$$

$$A_O(T)=0,A_F(T)=0$$

$$\lambda_4(T),\lambda_5(T)\rightarrow free$$

Kuhn-Tucker conditions

Control variables

$$\frac{\partial\tilde{L}}{\partial s_j}\le 0,\frac{\partial\tilde{L}}{\partial s_j}s_j=0$$

$$\frac{\partial\tilde{L}}{\partial \tau_j}\le 0,\frac{\partial\tilde{L}}{\partial \tau_j}\tau_j=0$$

$$\frac{\partial\tilde{L}}{\partial \xi}\le 0,\frac{\partial\tilde{L}}{\partial \xi}\xi=0$$

$$\frac{\partial\tilde{L}}{\partial NX_1}=0$$

Lagrange multipliers

$$\mu_{1j}(1-s_j)=0,\mu_{2j}(1-\tau_j)=0,$$

$$\mu_3(1-\xi)=0,\mu_{4j}I_j=0,\mu_{5j}E_j=0$$

$$\mu_{1j}\ge 0,\mu_{2j}\ge 0,\mu_3\ge 0,\mu_{4j}\ge 0,\mu_{5j}\ge 0,$$

$\mu_{ij}=0$ when internal solution is assumed.

→ Control variables can be solved numerically.

Specifications for numerical analysis

Production function

$$Y_1(t)=K_1(t)^{a_1}H_1(t)^{a_2}L_1(t)^{a_3}$$

$$Y_2(t)=K_2(t)^{a_1}H_2(t)^{a_2}L_2(t)^{a_3}$$

Both countries share the same Cobb-Douglas function.

Prameter α satisfies Σα=1 (Linearly homogeneous)

Utility function

$$u(c_{Hj}(t))=\ln(c_{Hj}(t))$$

$$u(c_{Lj}(t))=\ln(c_{Lj}(t))$$

Efficiency of education

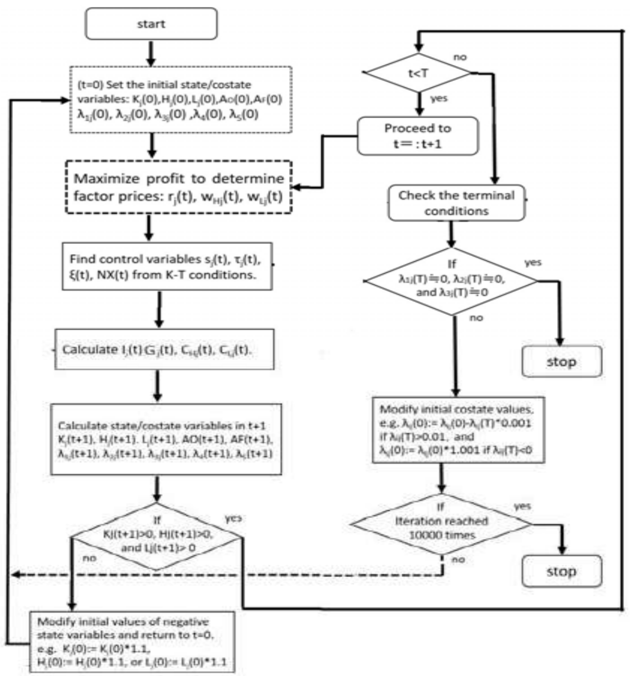
$$\Psi(G_j(t))=\varepsilon\ln(G_j(t)+1)$$

Terminal conditions

$$A_O(T)=0,A_F(T)=0$$

$$\lambda_1(T)=0,\lambda_2(T)=0,\lambda_3(T)=0\quad \lambda_4(T),\lambda_5(T)\rightarrow free$$

Outline of the iterative solution procedure



Terminal year=100

List of alternative values of parameters

Variable	North	South	# Cases
Production function	$(\alpha_1, \alpha_2, \alpha_3)$ = (0.4, 0.35, 0.25), (0.5, 0.3, 0.2), (0.6, 0.3, 0.1)		3
Speed of migration	0.5, 1.0, 1.5		3
Capital depreciation	0.04, 0.08, 0.12	0.035, 0.07, 0.105	9
Efficiency of education	0.25, 0.5, 0.75		3
Fertility rate	0.0071, 0.0142, 0.0213	0.0223, 0.0446, 0.0669	81
Mortality rate	0.01305, 0.00435, 0.0087	0.0083, 0.0166, 0.0249	
Weight of welfare	1.0	0.45, 0.9, 1.35	3
Subjective discount	0.01, 0.02, 0.03		3
Interest rate	0.01, 0.02, 0.03		3
Asset share of skilled	0.6, 1.2, 1.8	0.75, 1.5, 2.25	9
Initial values	$K_1(0)$=1200, $H_1(0)$=40, $L_1(0)$=75	$K_2(0)$=90, $H_2(0)$=20, $L_2(0)$=100	

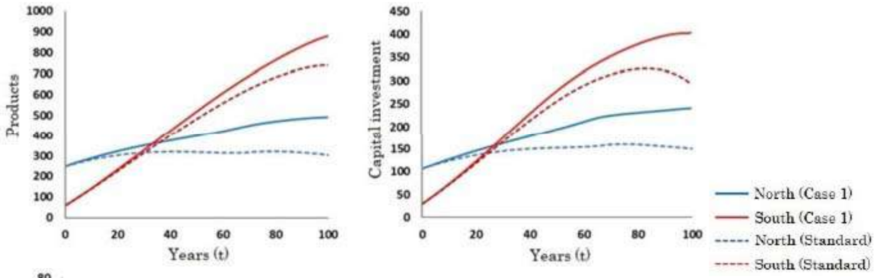
- Each parameter is assigned three values, and those in red represent the standard case.
- Standard values of capital depreciation, fertility, and mortality rates are based on the observed data.
- When one parameter is diverted from the standard case at a time, the sum of 117 cases are generated.

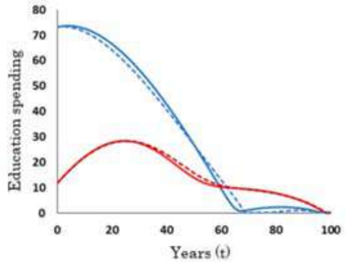
Cases reached convergence

	North	South	North	South	L(0) in North
Case	Fertility rate		Mortality rate		
1	0.0142	0.0223	0.0087	0.0166	
2	0.0071	0.0223	0.0087	0.0166	100
3	0.0071	0.0223	0.01305	0.0166	110
4	Speed of migration η=0.5				
5	Interest rate r_p=0.03				

- In cases 2 and 3, the initial unskilled workers in *North* are modified to the values comparable to those in *South* in order to reach the solution.
- In practice, it is not possible to change the number of existing workers.
- The current calculations do not consider the terminal conditions concerning the *cumulative debts*.
- Even though, only cases 1, 4, and 5 are deemed solvable besides the standard case, under the *present numerical procedure.*

Comparative dynamics: Decrease in mortality rate in the North (Case 1)

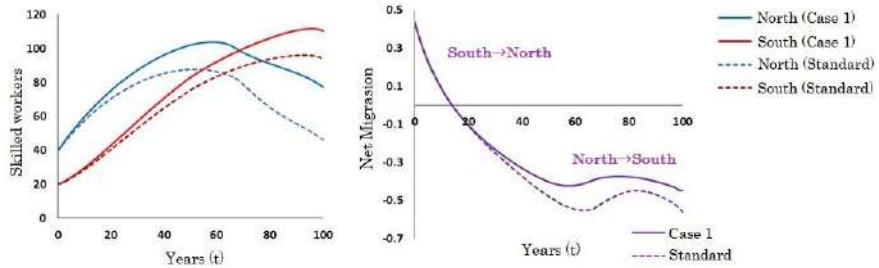


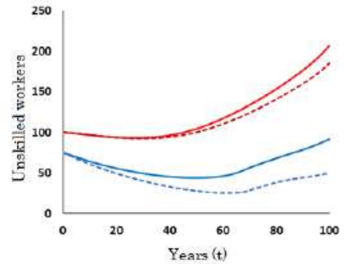


Production: Increases in both countries
Physical investment: Increases in both countries
Education spending: Similar to the standard case

Decrease in death in the North will contribute to production increase, which would expand the fund for physical investment (in both countries).

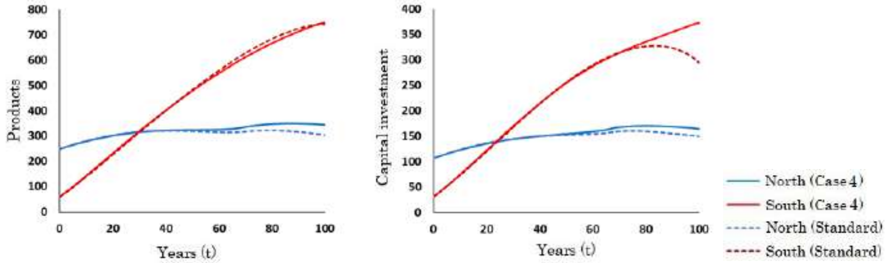
Comparative dynamics: Decrease in mortality rate in the North (Case 1)

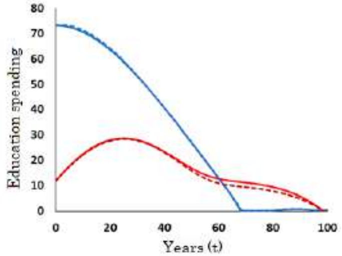




- Both skilled and unskilled workers increase in both countries.
- Migrants from North to South will decrease.
- Skilled workers will increase despite unchanged educational spending.

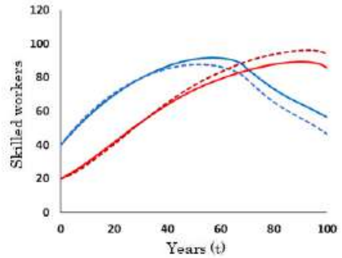
Comparative dynamics: Decrease in the speed of migration (Case 4)

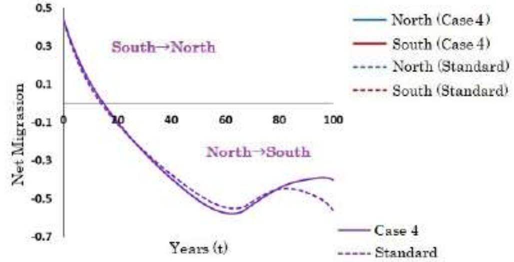


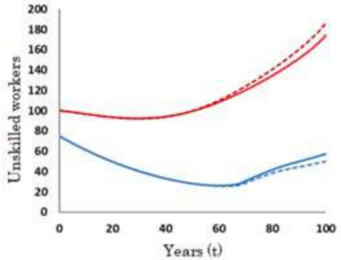


Production: Increases in North, but decreases in South.
Physical investment: Increases after year 80 in both countries.
Educational spending: Similar to the standard case.

Comparative dynamics: Decrease in the speed of migration (Case 4)

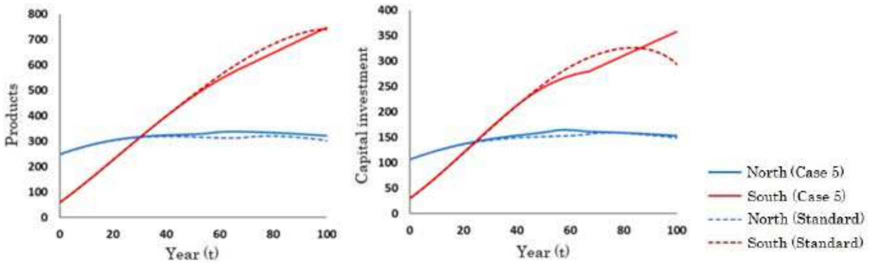


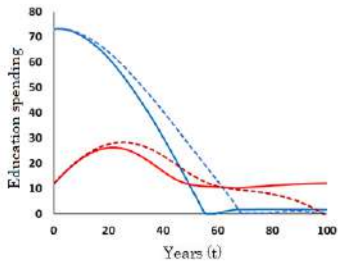




- Both skilled and unskilled workers: Increase in North, but decrease in South.
- Outmigration from North will decrease after year 80.
- Human capital in North will increase despite unchanged educational spending.

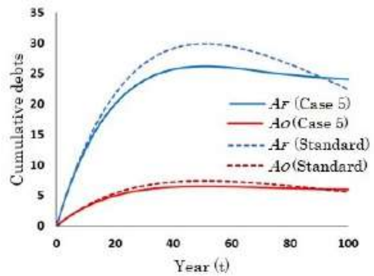
Comparative dynamics: Increase in interest rate (Case 5)

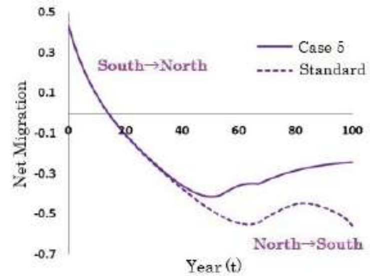




Production: Increases in North, but decreases in South.
Physical investment: Decreases in South between years 50 and 80, but increases later.
Educational spending: Decreases in North, but decrease in South is seen between years 20 and 60.
Increase in interest rate affects educational spending in both countries, but no essential change in production or physical investment.

Comparative dynamics: Increase in interest rate (Case 5)





- FDI related cumulative debts decrease until year 90, but increase in the last years.
- No essential change is found with ODA related cumulative debts.
- Outmigration from North will decrease after year 40.

Increase in interest rate naturally affects the time path of debt services.

Concluding remarks

An optimal control problem for the world comprising two countries and considering two types of workers is formulated.

Research questions:

- How the population declining country can coexist with the population increasing country?
- Is it possible for the former to survive with debt services from the latter?

Model simplifications:

- A dynasty model; birth and death are considered, but they occur as a Markov process, regardless of age.→ OLG setting is an alternative.
- Workers hold only domestic shares to avoid migration hysteresis problem.
- No uncertainty, which is important in international financial obligations, is considered.
- The *world government problem* is the simplest way to solve this kind of problems, but the solutions can serve as the reference to other (less efficient) solution procedures; e.g. class of dynamic games.

Concluding remarks

Numerical analyses seem only practical approach to the model; Iterative solution procedure is developed.

Find sets of parameters with which the admissible path exist:

- Model includes 8 parameters, such as the fertility/mortality rates, which must be specified for each country, and 6 parameters, such as interest and subjective discount rates, which are common to both countries.
- In this study, a sum of 117 cases, each of which reflects alteration of one or two parameters from the standard case, are tested.
- Only three of them can be solved, but still difficult to meet the terminal condition of paying off previous debts.

The transient solutions seem to exist for fairly limited sets of parameters.

- The numerical procedure must be reviewed to reduce the non-solvable case caused by technical reason. → Use of GAMS?
- Extensive tests must be conducted to find the region of parameters, for which the population declining economy becomes sustainable.